UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**MATH3801**
ASSESSMENT PATTERN	:	**MATH3801A**
MODULE NAME	:	**Logic**
DATE	:	**26-May-10**
TIME	:	**14:30**
TIME ALLOWED	:	**2 Hours 0 Minutes**

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TURN OVER

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All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) State the algorithm for restoring brackets to a 'slang' formula. Express the following as $\mathcal{L}$(practical) formulas:

   (i) $\mathrm{Ax} \to \exists \mathrm{yBy} \to \mathrm{Cx} \wedge \mathrm{Dz}$,

   (ii) $(\alpha \to \beta \vee \gamma) \wedge \alpha \to \beta \wedge (\alpha \to \gamma)$.

   (b) Describe how the 'free' variables are defined in an $\mathcal{L}$(minimal) formula. Write down $\alpha(\mathrm{x}/\mathrm{y})$ and $\alpha(\mathrm{x}/\mathrm{y})_S$, where α is the formula $\mathrm{Ax} \to \exists \mathrm{xLxy} \to \mathrm{Cx}$.

2. Write down the propositional rules for a tableau. If α, β, γ and δ are $\mathcal{L}$ formulas, use the tableau method to determine which, if any, of the following are tautologies:

   (a) $\neg\alpha \wedge \alpha \to \beta$,

   (b) $\neg\alpha \wedge \alpha \to \beta \vee \gamma \to \alpha$,

   (c) $(\alpha \to \beta \to \gamma) \to \neg\alpha \vee \beta \to \neg\alpha \vee \gamma$.

3. Write down the quantifier rules for a tableau. Discuss the following entailments using the tableau method.

   (a) $\models \forall \mathrm{x}(\mathrm{Lxx} \to \exists \mathrm{yLyx})$,

   (b) $\{\forall \mathrm{x} \neg \mathrm{Lxx}, \forall \mathrm{x} \forall \mathrm{z}(\exists \mathrm{y}(\mathrm{Lxy} \wedge \mathrm{Lyx}) \to \mathrm{Lxz})\} \models \forall \mathrm{x} \forall \mathrm{y} \neg(\mathrm{Lxy} \wedge \mathrm{Lyx})$.

4. (a) Give the definition for a Hintikka set of $\mathcal{L}$(minimal) formulas. If X is a Hintikka set which contains the formula $\neg\forall x \rightarrow \neg\forall z Lzx\ Hx$, show that it cannot contain the formula $\forall z \forall y \neg Lzy$.

   (b) Suppose that a semantic tableau for a set X of $\mathcal{L}$ formulas never terminates, show that there must be at least one infinite open branch. Without going into detail or giving more than a minimal justification for your answers, what can you say about the formulas in this branch? What can you say about X?

5. (a) What is meant by the following: 'An effective positive test', 'The Halting Problem'.

   Define the function '$BB(x)$'. Show that, if there were an effective positive test for non-halting Turing Machines, $BB(x)$ would be a computable function.

   (b) Define the *algorithmic complexity* of a binary string relative to an ideal computer. Give an upper bound for the number of strings of complexity less than n. Hence show that, for suitably large n, 99.9% of binary strings cannot be generated by strings much shorter than themselves. By considering integers defined by the expression $\text{Min}\{x : \text{complexity of } x > k\}$, for some suitably large k, show that, for any automated theorem prover describing the integers, there will be true statements of the form $\exists x Q(x)$ but no x can be found for which $Q(x)$ holds.

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